NAG Toolbox for MATLAB

f01qj

1 Purpose

f01qj finds the RQ factorization of the real m by n ($m \leq n$) matrix A, so that A is reduced to upper triangular form by means of orthogonal transformations from the right.

2 Syntax

```
[a, zeta, ifail] = f01qj(a, 'm', m, 'n', n)
```

3 Description

The m by n matrix A is factorized as

$$A = (\,R \quad 0\,)P^{\mathrm{T}} \quad \text{when} \quad m < n,$$

$$A = RP^{\mathrm{T}} \qquad \text{when} \quad m = n,$$

where P is an n by n orthogonal matrix and R is an m by m upper triangular matrix.

P is given as a sequence of Householder transformation matrices

$$P = P_m \ldots P_2 P_1,$$

the $(m-k+1)$th transformation matrix, P_k, being used to introduce zeros into the kth row of A. P_k has the form

$$P_k = I - u_k u_k^{\mathrm{T}},$$

where

$$u_k = \begin{pmatrix} w_k \\ \zeta_k \\ 0 \\ z_k \end{pmatrix},$$

ζ_k is a scalar, w_k is an $(k-1)$ element vector and z_k is an $(n-m)$ element vector. u_k is chosen to annihilate the elements in the kth row of A.

The vector u_k is returned in the kth element of **zeta** and in the kth row of **a**, such that ζ_k is in $\mathbf{zeta}(k)$, the elements of w_k are in $\mathbf{a}(k,1),\ldots,\mathbf{a}(k,k-1)$ and the elements of z_k are in $\mathbf{a}(k,m+1),\ldots,\mathbf{a}(k,n)$. The elements of R are returned in the upper triangular part of **a**.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H 1965 *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda,**$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1,\mathbf{m})$

The second dimension of the array must be at least $\max(1,\mathbf{n})$

The leading m by n part of the array **a** must contain the matrix to be factorized.

5.2 Optional Input Parameters

1: **m – int32 scalar**

m, the number of rows of the matrix A.

When $\mathbf{m} = 0$ then an immediate return is effected.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq \mathbf{m}$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda

5.4 Output Parameters

1: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by m upper triangular part of **a** will contain the upper triangular matrix R, and the m by m strictly lower triangular part of **a** and the m by $(n - m)$ rectangular part of **a** to the right of the upper triangular part will contain details of the factorization as described in Section 3.

2: **zeta(∗) – double array**

Note: the dimension of the array **zeta** must be at least $\max(1, \mathbf{m})$.

zeta(k) contains the scalar ζ_k for the $(m - k + 1)$th transformation. If $P_k = I$ then **zeta**$(k) = 0.0$, otherwise **zeta**(k) contains ζ_k as described in Section 3 and ζ_k is always in the range $\left(1.0, \sqrt{2.0}\right)$.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= -1$

On entry, **m** < 0,
or **n** $<$ **m**,
or **lda** $<$ **m**.

7 Accuracy

The computed factors R and P satisfy the relation

$$(R \quad 0)P^{\mathrm{T}} = A + E,$$

where

$$\|E\| \leq c\epsilon\|A\|,$$

ϵ is the ***machine precision*** (see x02aj), c is a modest function of m and n, and $\|.\|$ denotes the spectral (two) norm.

8 Further Comments

The approximate number of floating-point operations is given by $2m^2(3n-m)/3$.

The first k rows of the orthogonal matrix P^{T} can be obtained by calling f01qk, which overwrites the k rows of P^{T} on the first k rows of the array **a**. P^{T} is obtained by the call:

```
[a, ifail] = f01qk('Separate', m, k, a, zeta);
```

WORK must be a $\max(m-1, k-m, 1)$ element array. If K is larger than M, then **a** must have been declared to have at least K rows.

9 Example

```
a = [2, 2, 1.6, 2, 1.2;
     2.5, 2.5, -0.4, -0.5, -0.3;
     2.5, 2.5, 2.8, 0.5, -2.9];
[aOut, zeta, ifail] = f01qj(a)
```

```
aOut =
   -3.1446   -1.0705   -2.2283    0.6333    0.7619
    0.5277   -2.8345   -2.2283   -0.1662    0.0945
    0.3766    0.3766   -5.3852    0.0753   -0.4368
zeta =
    1.0092
    1.2981
    1.2329
ifail =
           0
```